TOPIC 1

Circles and Ratio

Dropping something into water causes a series of ripples to expand from the point of impact, forming concentric circles.

Module 1: Thinking Proportionally

TOPIC 1: CIRCLES AND RATIO

In this topic, students learn formulas for the circumference and area of circles and use those formulas to solve mathematical and real-world problems. To fully understand the formulas, students develop an understanding of the irrational number pi (π) as the ratio of a circle's circumference to its diameter. Throughout the topic, students practice applying the formulas for the circumference and area of a circle, often selecting the appropriate formula. Finally, students practice applying the formulas by using them to solve a variety of problems, including calculating the area of composite figures.

Where have we been?

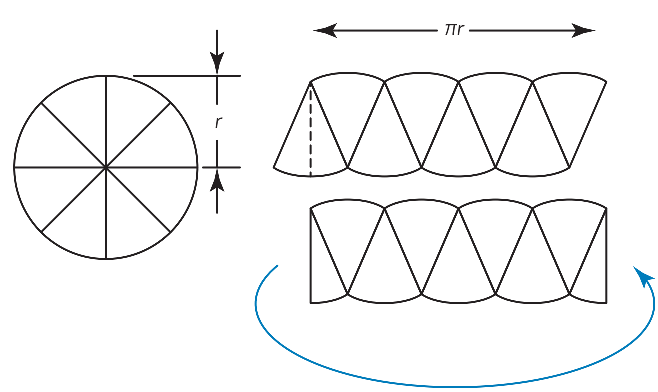
Throughout elementary school, students used and labeled circles and determined the perimeters of shapes formed with straight lines. In grade 6, students worked extensively with ratios and ratio reasoning. To begin this topic, students draw on these experiences as they use physical tools to investigate a constant ratio, pi.

Where are we going?

This early review of and experience with ratios prepares students for future lessons where they will move from concrete representations and reasoning about ratios and proportions to more abstract and symbolic work with solving proportions and representing proportional relationships. In future grades, students will use the circumference and area formulas of a circle to calculate surface areas and volumes of cylinders and composite three-dimensional shapes that include circles.

Modeling the Area of a Circle Using Wedges

Divide a circle into a large number of equal-sized wedges. Laying these wedges as shown, you can see that they approximate a rectangle with a length of πr and a height of r. The more wedges are added, the closer the figure will be to an exact rectangle. So, the rectangle of wedges, and thus, the circle, each has an area of πr^2.



Myth: "I don't have the math gene."

Let's be clear about something. There isn't ***a*** gene that controls the development of mathematical thinking. Instead, there are probably ***hundreds*** of genes that contribute to our ability to reason mathematically. Moreover, a recent study suggests that mathematical thinking arises from the ability to learn a language. Given the right input from the environment, children learn to speak without any formal instruction. They can learn number sense and pattern recognition the same way.

To further nurture your child's mathematical growth, attend to the learning environment. You can think of it as providing a nutritious mathematical diet that includes discussing math in the real world, offering the right kind of encouragement, being available to answer questions, allowing your student to struggle with difficult concepts, and giving them space for plenty of practice.

#mathmythbusted

Talking Points

You can further support your student's learning by asking questions about the work they do in class or at home. Your student is learning to think flexibly about mathematical relationships involving multiplication, area, and number properties.

Questions to Ask

- How does this problem look like something you did in class?
- Can you show me the strategy you used to solve this problem? Do you know another way to solve it?
- Does your answer make sense? How do you know?
- Is there anything you don't understand? How can you use today's lesson to help?

Key Terms

radius

The radius of a circle is a line segment formed by connecting a point on the circle and the center of the circle.

diameter

The diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.

circumference

The circumference of a circle is the distance around the circle. The circumference is calculated using the formula $C = \pi d$.

pi

The number pi (π) is the ratio of the circumference of a circle to its diameter.

Pi: The Ultimate Ratio

1

Exploring the Ratio of Circle Circumference to Diameter

WARM UP

Scale up or down to determine an equivalent ratio.

1. $\frac{18 \text{ miles}}{3 \text{ hours}} = \frac{?}{1 \text{ hour}}$

2. $\frac{\$750}{4 \text{ days}} = \frac{?}{1 \text{ day}}$

3. $\frac{12 \text{ in.}}{1 \text{ ft}} = \frac{?}{5 \text{ ft}}$

4. $\frac{48 \text{ oz}}{3 \text{ lb}} = \frac{?}{1 \text{ lb}}$

LEARNING GOALS

- Identify pi (π) as the ratio of the circumference of a circle to its diameter.
- Construct circles using a compass and identify various parts of circles.
- Know and write the formula for the circumference of a circle, and use the formula to solve problems.

KEY TERMS

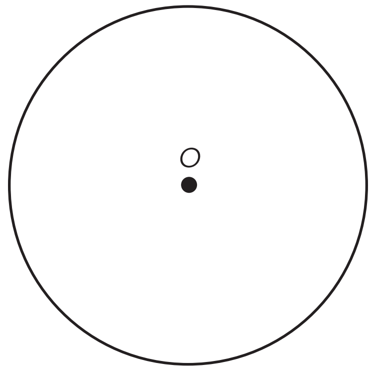
- congruent
- circle
- radius
- diameter
- circumference
- pi

You have learned about ratios. How can you use ratios to analyze the properties of geometric figures such as circles?

Getting Started

Across and Around

A circle is shown with a point drawn at the center of the circle. The name of the point is *O*, so let's call this Circle *O*.



1. Analyze the distance around the circle.

a. Use a string and a centimeter ruler to determine the distance around the circle.

b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

2. Draw a line from a point on the circle to the center of the circle, point *O*.

a. Measure your line using your centimeter ruler.

b. How does your measurement compare to your classmates' measurements? Summarize the similarities and differences.

ACTIVITY 1.1

Analyzing the Parts of a Circle

Everyone can identify a circle when they see it, but defining a circle is a bit harder. Can you define a circle without using the word *round*? Investigating how a circle is formed will help you mathematically define a circle.

1. Follow the given steps to investigate how a circle is formed.

Step 1: In the space provided, draw a point and label the point *A*.

Step 2: Use a centimeter ruler to locate and draw a second point that is exactly 5 cm from point *A*. Label this point *B*.

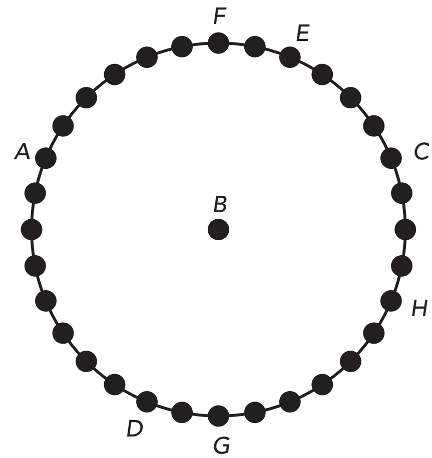
Step 3: Locate a third point that is exactly 5 cm from point *A*. Label this point *C*.

Step 4: Repeat this process until you have drawn at least ten distinct points that are each exactly 5 cm from point *A*.

2. How many other points could be located exactly 5 cm from point *A*? How would you describe this collection of points in relation to point *A*?

3. Define the term *circle* without using the word *round*.

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant. Circles are named by their center point.



4. Use the circle shown to answer each question.

a. Name the circle.

The **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is the diameter of the circle. The **diameter** of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point. The distance around a circle is called the **circumference** of the circle.

b. Identify a radius of the circle.

c. Identify a diameter of the circle.

d. Are all radii of this circle the same length? Explain your reasoning.

5. What is the relationship between the length of a radius and the length of a diameter?

ACTIVITY 1.2

Measuring the Distance Around a Circle

Let's explore circles. Use circles *A*, *B*, *D*, *E*, and *O* provided at the end of the lesson. Circle *O* is the same as the circle from the activity *Across and Around*.

1. Use a string and a centimeter ruler to measure the distance from a point on the circle to the center and the distance around each circle. Record your measurements in the table. In the last column, write the ratio of *Circumference* : *Diameter* in fractional form.

Circle	Circumference	Radius	Diameter	$\frac{\text{Circumference}}{\text{Diameter}}$
Circle *A*				
Circle *B*				
Circle *O*				
Circle *D*				
Circle *E*				

2. Average the ratios recorded for $\frac{\text{Circumference}}{\text{Diameter}}$. What is the approximate ratio for the circumference to the diameter for the set of circles? Write the approximate ratio as a fraction and as a decimal.

3. How does your answer to Question 2 compare to your classmates' answers?

4. Average all of your classmates' answers to Question 3. Write the approximate ratio of circumference to the diameter as a fraction and as a decimal.

NOTES

ACTIVITY 1.3

The Circumference Formula

The number **pi** (π) is the ratio of the circumference of a circle to its diameter. That is pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$.

1. Use this information to write a formula for the circumference of a circle, where *d* represents the diameter of a circle and *C* represents the circumference of a circle.

2. Rewrite the formula for the circumference of a circle, where *r* represents the radius of a circle and *C* represents the circumference of a circle.

3. Use different representations for π to calculate the circumference of a circle.

a. Calculate the circumference of a circle with a diameter of 4.5 centimeters and a circle with a radius of 6 inches. Round your answer to the nearest ten-thousandths, if necessary.

Value for π	d = 4.5 centimeters	r = 6 inches
π		
Use the π Key on a Calculator		
Use 3.14 for π		
Use $\frac{22}{7}$ for π		

b. Compare your circumference calculations. How do the different values of π affect your calculations?

4. Use the circumference of a circle formula to determine each unknown. Use 3.14 for π.

   a. Compute the diameter of the circle with a circumference of 65.94 feet.

   b. Compute the radius of the circle with a circumference of 109.9 millimeters.

5. What is the minimum amount of information needed to compute the circumference of a circle?

NOTES

TALK the TALK

Twice

Use what you have learned to compare circles by their characteristics.

1. Draw each circle.

a. radius length of 3 centimeters

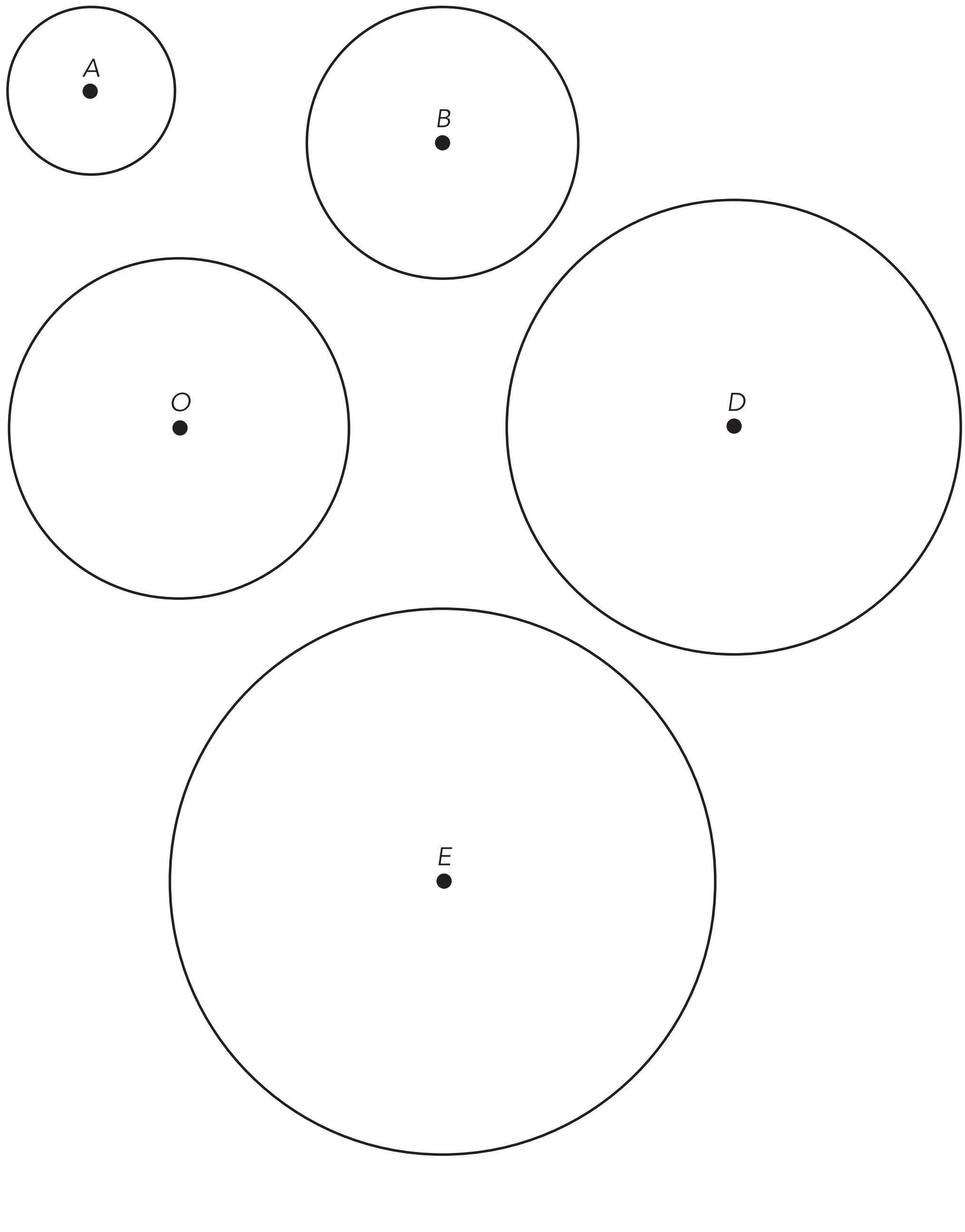
b. diameter length of 3 centimeters

2. Describe the similarities and differences between your two circles.

3. Describe the relationship between the circumferences of the two circles.

4. Describe the circumference-to-diameter ratio of all circles.

Measuring the Distance Around a Circle



Assignment

Write

Define each term in your own words.

1. circle
2. radius
3. diameter
4. pi

Remember

The circumference of a circle is the distance around the circle. The formulas to determine the circumference of a circle are $C = \pi d$ or $C = 2\pi r$, where d represents the diameter, r represents the radius, and π is a constant value equal to approximately 3.14 or $\frac{22}{7}$.

The constant pi (π) represents the ratio of the circumference of a circle to its diameter.

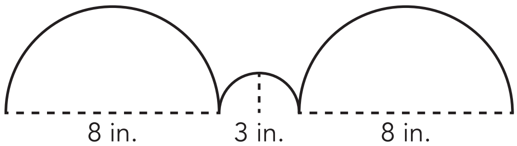
Practice

Answer each question. Use 3.14 for π. Round your answer to the nearest hundredth, if necessary.

1. Although she's only in middle school, Tameka loves to drive go-carts! Her favorite place to drive go-carts, Driver's Delight, has 3 circular tracks. Track 1 has a radius of 60 feet. Track 2 has a radius of 85 feet. Track 3 has a radius of 110 feet.
   a. Compute the circumference of Track 1.
   b. Compute the circumference of Track 2.
   c. Compute the circumference of Track 3.
   d. Driver's Delight is considering building a new track. They have a circular space with a diameter of 150 feet. Compute the circumference of the circular space.
2. Tameka wants to build a circular go-cart track in her backyard.
   a. If she wants the track to have a circumference of 150 feet, what does the radius of the track need to be?
   b. If she wants the track to have a circumference of 200 feet, what does the radius of the track need to be?
   c. If she wants the track to have a circumference of 400 feet, what does the diameter of the track need to be?

Stretch

A rope is arranged using three semi-circles to form the pattern shown. Determine the length of the rope.



Review

1. Ethan and Corinne are training for a marathon.
   a. Corinne runs 13.5 miles in 2 hours. What is her rate?
   b. Ethan wants to run the 26.2 miles of the marathon in 4.5 hours. At about what rate will he have to run to reach this goal? Round to the nearest tenth.
2. Fifteen seventh graders were randomly selected to see how many pushups in a row they could do. Their data are shown.
   45, 40, 36, 38, 42, 48, 40, 40, 70, 45, 42, 43, 48, 36
   a. Determine the mean of this data set.
   b. Determine the median of this data set.
3. Convert each measurement.
   a. $4\frac{1}{2}$ pounds = ____ oz
   b. 22.86 cm = ____ in.

That's a Spicy Pizza!

2

Area of Circles

WARM UP

Determine a unit rate for each situation.

1. $38.40 for 16 gallons of gas

2. 15 miles jogged in 3.75 hours

3. $26.99 for 15 pounds

LEARNING GOALS

- Give an informal derivation of the relationship between the circumference and area of a circle and use the area formula to solve problems.
- Decide whether circumference or area is an appropriate measure for a problem situation.
- Calculate unit rates associated with circle areas.

KEY TERM

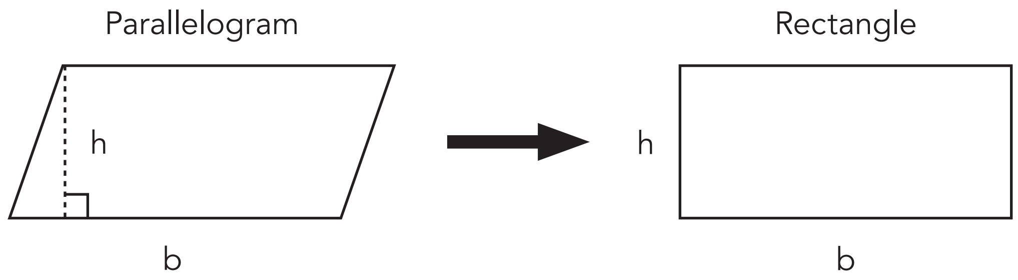
- unit rate

You have learned about the different parts and measures of a circle, including radius, diameter, and circumference. How can you use the parts of a circle to determine the area of a circle?

Getting Started

What Changed? What Stayed the Same?

The length of the base and height are the same in the parallelogram and rectangle shown.



1. How could you rearrange the parallelogram to create the rectangle?

2. What is the area of each figure?

ACTIVITY
2.1

Deriving the Area Formula

In the last lesson you derived formulas for the distance around a circle. In this lesson you will investigate the space within a circle. Use the circle at the end of the lesson that is divided into 4, 8, and 16 equal parts.

1. Follow the steps to decompose the circle and compose it into a new figure.

a. First, cut the circle into fourths and arrange the parts side by side so that they form a shape that looks like a parallelogram.

b. Then cut the circle into eighths and then sixteenths. Each time, arrange the parts to form a parallelogram.

2. Analyze the parallelogram you made each time.

a. How did the parallelogram change as you arranged it with the smaller equal parts of the same circle?

b. What would be the result if you built the parallelogram out of 40 equal circle sections? What about 100 equal circle sections?

c. Represent the approximate base length and height of the parallelogram in terms of the radius and circumference of the circle.

NOTES

d. Use your answers to part (c) to determine the formula for the area of the parallelogram.

e. How does the area of the parallelogram compare to the area of the circle?

f. Write a formula for the area of a circle.

3. Use different representations for π to calculate the area of a circle.

a. Calculate the area of each circle with the given radius. Round your answers to the nearest ten-thousandths, if necessary.

Value for π	$r = 6$ units	$r = 1.5$ units	$r = \frac{1}{2}$ units
π			
Use the π Key on a Calculator			
Use 3.14 for π			
Use $\frac{22}{7}$ for π			

b. Compare your area calculations for each circle. How do the different values of π affect your calculations?

4. Suppose the ratio of radius lengths of two circles is 1 unit to 2 units.

   a. What is the ratio of areas of the circles? Experiment with various radius lengths to make a conclusion.

   b. If the length of the radius of a circle is doubled, what effect will this have on the area?

ACTIVITY

2.2

Circumference or Area

Circle Formulas

$C = \pi d$, or $2\pi r$

$A = \pi r^2$

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. When solving problems involving circles, it is important to think about what you are trying to determine.

1. A city park has a large circular garden with a path around it. The diameter of the garden is 60 feet.

a. Gina likes to walk along the circular path during her lunch breaks. How far does Gina walk if she completes one rotation around the path?

b. Jason works for the City Park Department. He needs to spread plant food all over the garden. What is the area of the park he will cover with plant food?

2. Samantha is making a vegetable pizza. First, she presses the dough so that it fills a circular pan with a 16-inch diameter and covers it with sauce. What is the area of the pizza Samantha will cover with sauce?

3. Members of a community center have decided to paint a large circular mural in the middle of the parking lot. The radius of the mural is to be 11 yards. Before they begin painting the mural, they use rope to form the outline. How much rope will they need?

ACTIVITY 2.3

Unit Rates and Circle Area

Talarico's Pizza has a large variety of pizza sizes.

	Small	Medium	Large	X-Large	Enorme	Ginorme	Colossale
Diameter	10 in.	13 in.	16 in.	18 in.	24 in.	28 in.	36 in.
Slices	6	8	10	12	20	30	40
Cost	$6.99	$9.99	$12.99	$14.99	$22.99	$28.99	$54.99

Lina and Michael are trying to decide whether to get two pizzas or one Ginorme pizza. They ask themselves, "Which choice is the better buy?"

They each calculated a unit rate for the Ginorme pizza.

Recall that a **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

Lina

1 Ginorme: $\frac{\pi(14)^2}{28.99} = \frac{196\pi}{28.99} \approx 21.24$ square inches per dollar

The Ginorme gives you approximately 21.24 square inches of pizza per dollar.

Michael

1 Ginorme: $\frac{28.99}{14^2\pi} = \frac{28.99}{196\pi} \approx$ \$0.05 per square inch

The Ginorme costs approximately \$0.05 for each square inch of pizza.

1. Consider Lina's and Michael's work.

   a. Explain why Lina's and Michael's unit rates are different but still both correct.

   b. How would you decide which pizza was the better buy if you calculated the unit rate for each pizza using Lina's method versus Michael's method.

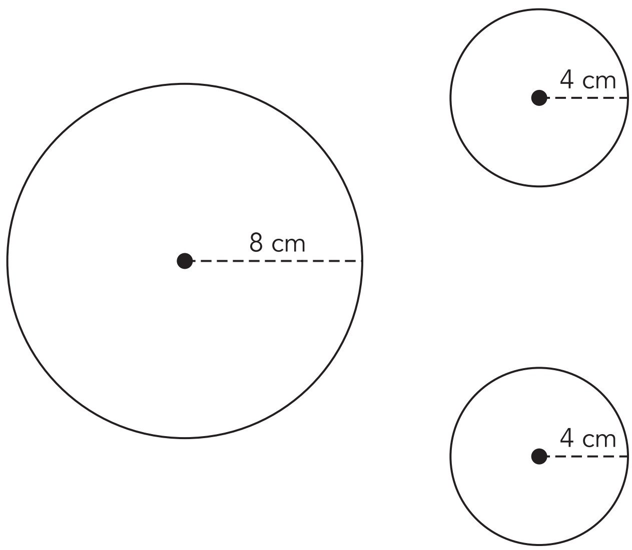
2. Which of the seven sizes of pizza from Talarico's Pizza is the best buy? Explain your answer.

NOTES

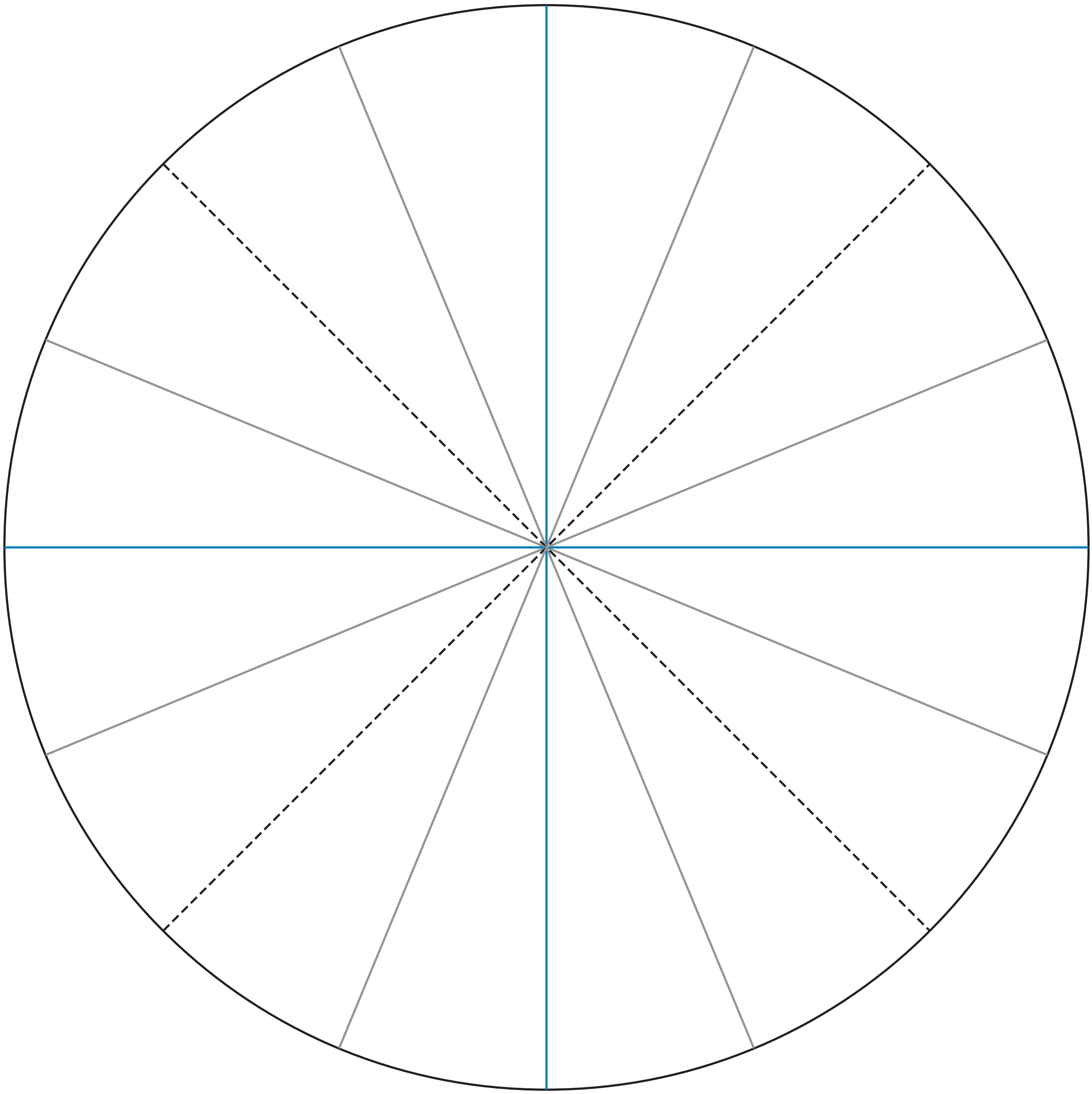
TALK the TALK

Go With the Flow

1. **Which pipe configuration can deliver more water to residents, one 8-cm pipe or two 4-cm pipes? Show your work and explain your reasoning.**



Circle Area Cutouts



Assignment

Write

Explain in your own words how to derive the formula for the area of a circle.

Remember

A formula for the area of a circle is $A = \pi r^2$.

Practice

Determine the area of the circle, given each measurement. Use 3.14 for π and round to the nearest hundredth.

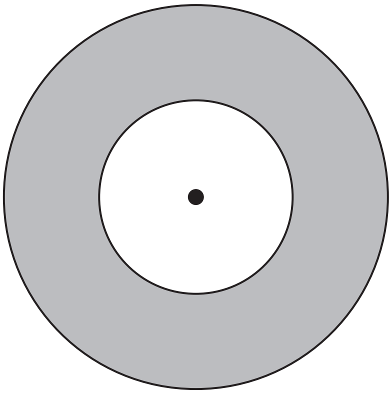
1. Diameter: 8 in.
2. Radius: 10 in.
3. Radius: 1.5 ft
4. Diameter: 8.8 yd
5. Diameter: $1\frac{3}{4}$ in.
6. Radius: $2\frac{1}{2}$ cm

Determine which pizza is the better buy in each situation.

7. The 10-inch diameter pizza for $8.99 or the 6-inch diameter pizza for $5.
8. The large 16-inch diameter pizza for $12.99 or the $26 X-large with a radius of 16 in.
9. The 12-inch diameter pizza for $12.50 or the 20-inch diameter pizza for $17.50.
10. The 4-inch radius pizza for $3 or the 8-inch radius pizza for $14.
11. Two 12-inch diameter pizzas for $12.98 or one large 14-inch diameter pizza for $7.99.
12. The 1-inch diameter pizza bite for $1 or the 10-inch diameter pizza for $10.

Stretch

The radius of the small circle is 0.5 millimeter. The area of the large circle is 28.26 square millimeters. Calculate the area of the shaded region.



Review

Determine the circumference of each circle, given its radius or diameter. Use 3.14 for π and round to the nearest tenth.

1. Radius: 4.5 cm
2. Diameter: 12 ft

Determine each unit rate.

3. 75 square feet of tile for $126
4. 420 miles in 6.5 hours

Compare the fractions in each pair using the symbol $>$, $<$, or $=$.

5. $\frac{3}{5}, \frac{2}{3}$
6. $\frac{6}{7}, \frac{8}{9}$

Circular Reasoning

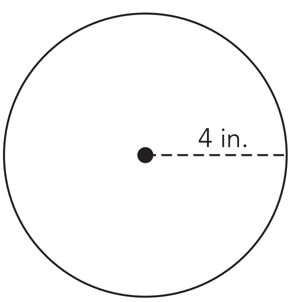
3

Solving Area and Circumference Problems

WARM UP

Determine the area of each circle. Use 3.14 for π.

1.



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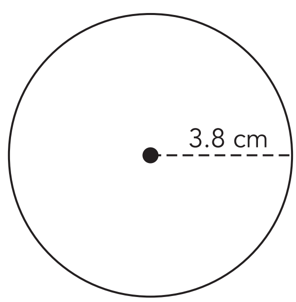

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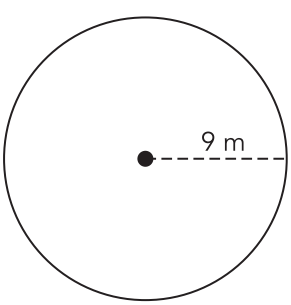

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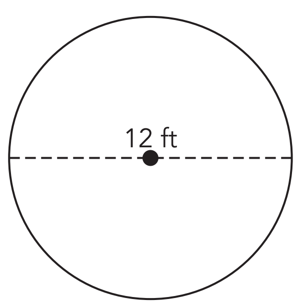


LEARNING GOALS

- Use the area and circumference formulas for a circle to solve problems.
- Calculate the areas of composite figures.

You encounter circles regularly in life. Now that you know how to calculate the circumference and area of circles, what kind of problems can you solve?

Getting Started

A Winning Formula

Suppose that the circumference of a circle is approximately 157 centimeters.

1. Describe a strategy you can use to solve for the area of the circle.

2. Solve for the area of the circle. Use 3.14 for π.

ACTIVITY
3.1

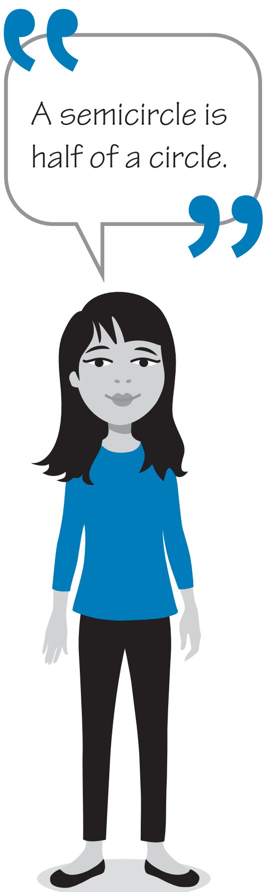
A Maximum Area Problem

A friend gave you 120 feet of fencing. You decide to fence in a portion of the backyard for your dog. You want to maximize the amount of fenced land.

1. **Draw a diagram, label the dimensions, and compute the maximum fenced area. Assume the fence is free-standing and you are not using any existing structure.**

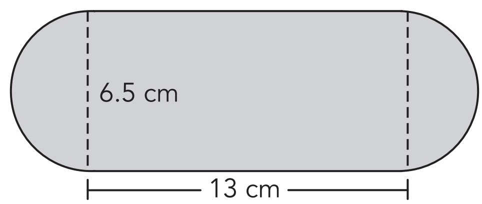
ACTIVITY
3.2

Composite Figure Problems

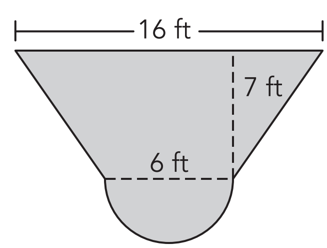


In previous grades you worked with composite figures made up of triangles and various quadrilaterals. Now that you know the area of a circle, you can calculate the area of more interesting composite figures.

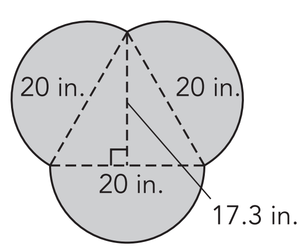
1. A figure is composed of a rectangle and two semicircles. Determine the area of the figure.



2. A figure is composed of a trapezoid and a semicircle. Determine the area of the figure.



3. A figure is composed of a triangle and three semicircles. Determine the area of the figure.

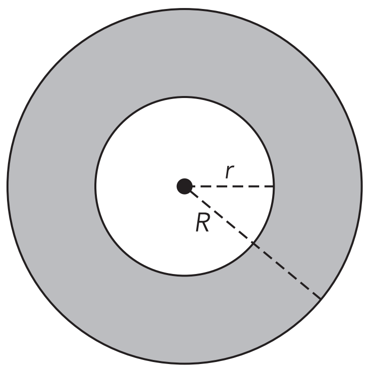


ACTIVITY
3.3

Shaded Region Problems

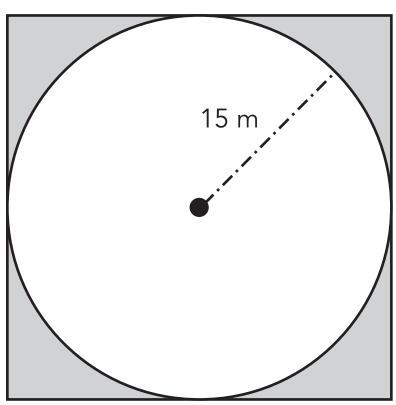
You have worked with composite figures by adding on areas. Now let's think about subtracting areas.

1. In the concentric circles shown, *R* represents the radius of the larger circle and *r* represents the radius of the smaller circle. Suppose that $R = 8$ centimeters and $r = 3$ centimeters. Calculate the area of the shaded region.



Concentric circles are circles with a common center. The region bounded by two concentric circles is called the annulus.

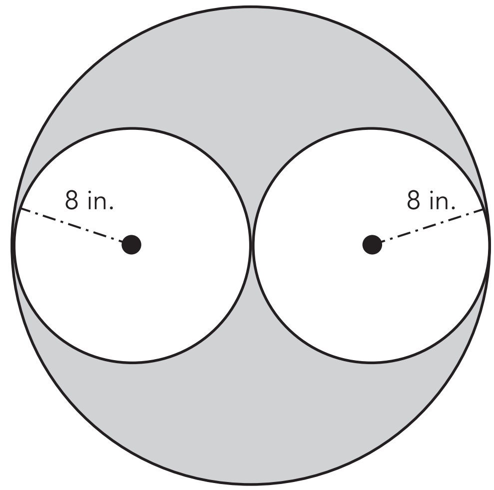
2. A circle is inscribed in a square. Determine the area of the shaded region.



When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.

NOTES

3. **Two small circles are drawn that touch each other, and both circles touch the large circle. Determine the area of the shaded region.**



4. **Jimmy and Matthew each said the area of the shaded region is about 402 square inches. Compare their strategies.**

Jimmy

Area of 1 small circle
$A \approx 3.14(8)^2$
$A \approx 3.14(64)$
$A \approx 200.96$

Area of 2 small circles
$A \approx 2(200.96)$
$A \approx 401.92$

Area of large circle
$A \approx (3.14)(16)^2$
$A \approx (3.14)(256)$
$A \approx 803.84$

Area of shaded region
$803.84 - 401.92 \approx 401.92$

The area of the shaded region is about 402 sq in.

Matthew

Area of 1 small circle
$A = \pi(8)^2$
$A = 64\pi$

Area of 2 small circles
$A = 2(64\pi)$
$A = 128\pi$

Area of large circle
$A = \pi(16)^2$
$A = 256\pi$

Area of shaded region
$256\pi - 128\pi = 128\pi$
$A = 128\pi$
$A \approx 402.12$

This means the area of the shaded region is about 402 sq in.

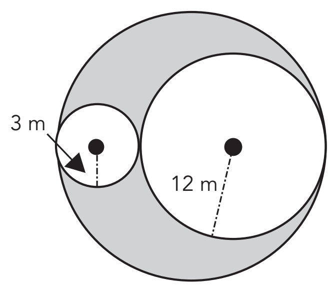
a. What did Jimmy and Matthew do the same?

b. What was different about their strategies?

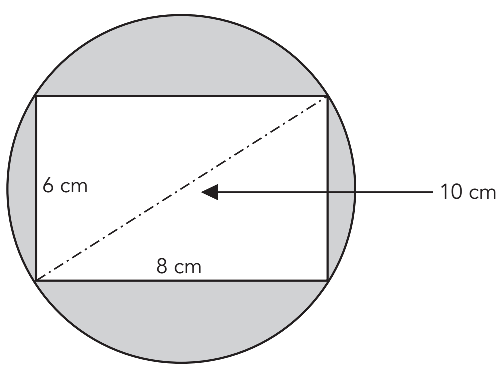
c. Which strategy do you prefer?

5. Determine the area of each shaded region.

a. One medium circle and one small circle touch each other, and each circle touches the large circle.

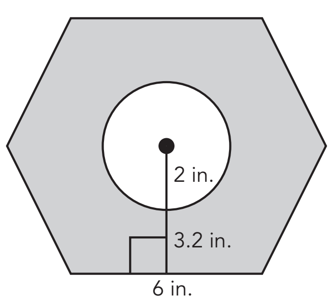


b. A rectangle is inscribed in a circle.



A rectangle is inscribed in a circle when all the vertices of the rectangle touch the circumference of the circle.

c. A circle is inside a regular hexagon.



NOTES

TALK the TALK

Rupert's Leash

Jamal loves his dog, Rupert. On sunny days, Jamal keeps Rupert on a 12-foot leash in the backyard. The leash is secured to a stake in the ground.

1. **Determine the diameter, circumference, and area of Rupert's play area. Use 3.14 for π.**

2. **Suppose Jamal wants to give Rupert a little more room to play. He uses a 15-foot leash instead of the 12-foot leash. What is the area of Rupert's play area now? Use 3.14 for π.**

Assignment

Write

Write the area and circumference formulas for circles.

Describe pi in terms of the area and radius of a circle. Describe pi in terms of the circumference and radius of a circle.

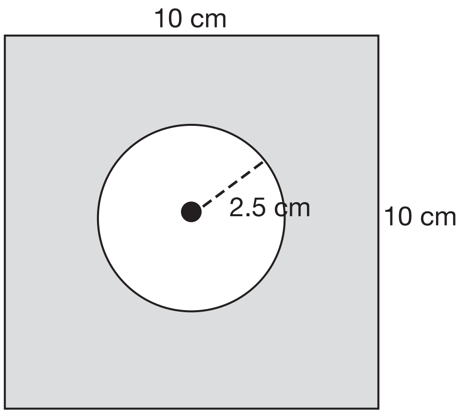
Remember

Given a specific length to form a perimeter or circumference, arranging that length into the shape of a circle provides the maximum area.

Practice

Calculate the area of the shaded region in each figure. Use 3.14 for π and round to the nearest tenth, if necessary.

1.



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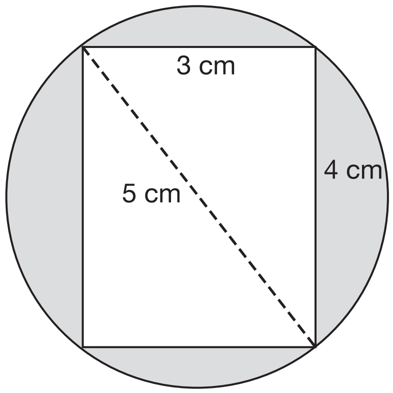

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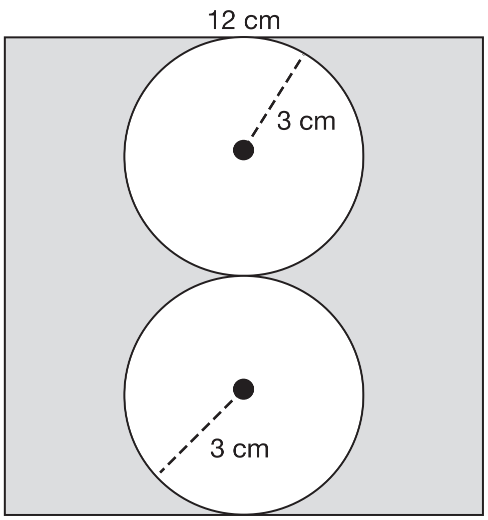

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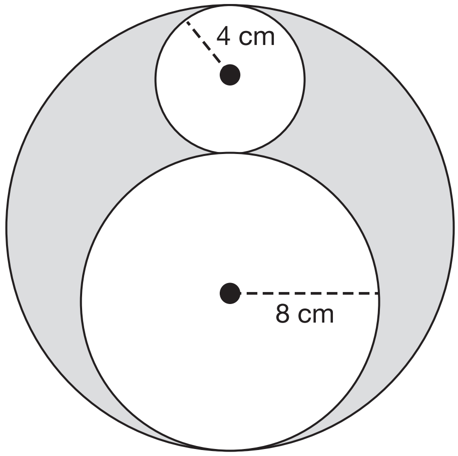
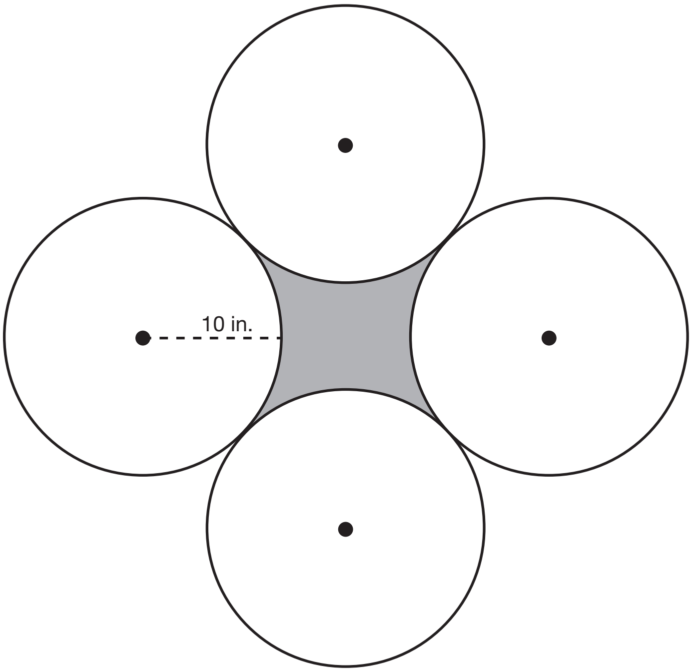


Stretch

1. Determine the area of the shaded region. All circles have the same radius of 10 inches.



Review

Solve each problem.

1. Jose is adding mulch to an existing round flower bed. The length of the rubber edging around the flower bed is 25.12 feet. What is the area that Jose needs to cover with mulch?
2. Nami is adding a mosaic pattern to the top of a small round table. The distance around the edge of the table top is 4.7 feet. What is the area that Nami needs to cover with the mosaic pattern?

Determine each area.

3. Area of a triangle with a base length of 4 in. and a height of 9 in.
4. Area of a parallelogram with a base length of 2.9 ft and a height of 5.5 ft.
5. Area of a trapezoid with a top base length of 6 cm, a bottom base length of 12 cm, and a height of 5 cm.

Write a unit rate for each ratio.

6. $\frac{28 \text{ cm}}{4 \text{ square feet}}$

7. $\frac{5.15 \text{ yd}}{5 \text{ square feet}}$

Circles and Ratio Summary

KEY TERMS

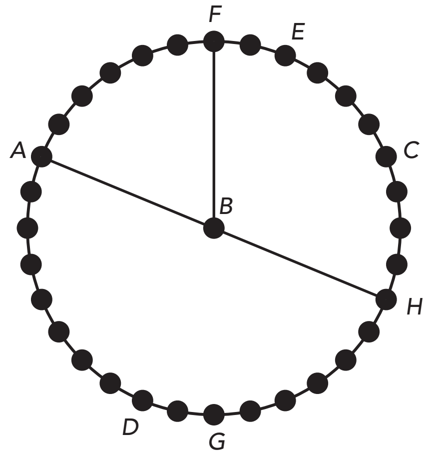
- congruent
- circle
- radius
- diameter
- circumference
- pi
- unit rate

LESSON 1

Pi: The Ultimate Ratio

A **circle** is a collection of points on the same plane equidistant from the same point. The center of a circle is the point from which all points on the circle are equidistant.

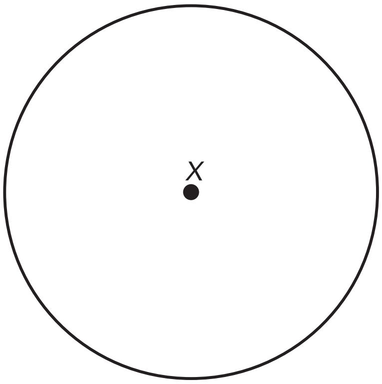
A **radius** of a circle is a line segment formed by connecting a point on the circle and the center of the circle. The distance across a circle through the center is a **diameter** of the circle. A diameter of a circle is a line segment formed by connecting two points on the circle such that the line segment passes through the center point.



Circles are named by their center point. For example, the circle shown is Circle *B*. A radius of Circle *B* is line segment *FB*. A diameter of Circle *B* is line segment *AH*.

The distance around a circle is called the **circumference** of the circle. The number **pi** (π) is the ratio of the circumference of a circle to its diameter. That is, pi $= \frac{\text{circumference of a circle}}{\text{diameter of a circle}}$, or $\pi = \frac{C}{d}$, where C is the circumference of the circle, and d is the diameter of the circle. The number π has an infinite number of decimal digits that never repeat. Some approximations used for the value π are 3.14 and $\frac{22}{7}$. You can use the ratio to write a formula for the circumference of a circle: $C = \pi d$.

Congruent means that it has the same shape and size. For example, Circle X is congruent to Circle B. If line segment AH on Circle B has a length of 10 centimeters, then the circumference of Circle X is $C = \pi(10)$ centimeters, or approximately 31.4 centimeters.

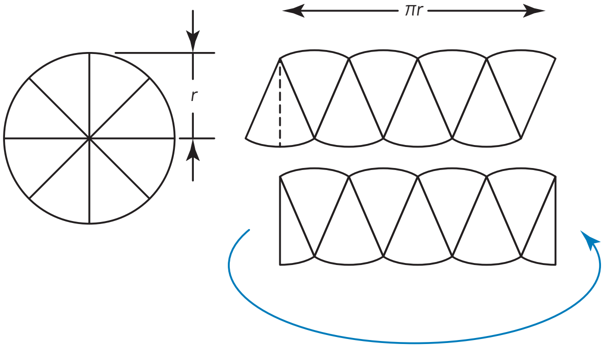


LESSON 2

That's a Spicy Pizza!

The circumference of a circle is the distance around the circle, while the area of a circle is the amount of space contained inside the circle. The formula for the area of a circle is $A = \pi r^2$.

The area formula for a circle can be derived by dividing a circle into a large number of equal-sized wedges. Laying these wedges as shown, you can see that they will form an approximate rectangle with a length of πr and a height of r.



A **unit rate** is a ratio of two different measures in which either the numerator or denominator is 1.

For example, a large pizza with a diameter of 18 inches costs $14.99. The rate of area to cost is $\frac{\pi \cdot 9^2}{14.99} = \frac{81\pi}{14.99}$. Using 3.14 for π, the unit rate is approximately 16.97 square inches per dollar. The unit rate of cost to area is $\frac{1}{16.97}$, or approximately $0.06 per square inch.

LESSON 3

Circular Reasoning

Given a specific length to form a perimeter or circumference, arranging that length into the shape of a circle provides the maximum area.

For example, suppose you have 176 feet of fencing to use to fence off a portion of your backyard for planting vegetables. You want to maximize the amount of fenced land. Calculate the maximum fenced area you will have.

The length of fencing you have will form the circumference of a circle. Use the formula for the circumference of a circle to determine the diameter of the fenced area.

$C = \pi d$

$176 = \pi d$

$56 \approx d$

If the diameter of the fenced area is about 56 feet, the radius is 28 feet. Use this information to calculate the area of the fenced land.

$A = \pi r^2$

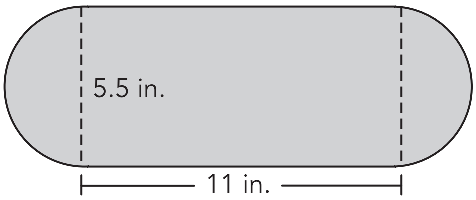
$A = \pi \cdot 28^2$

The maximum fenced area you will have is about 2461.76 square feet.

$A = 784\pi \approx 2461.76$

Many geometric figures are composed of two or more geometric shapes. These figures are known as composite figures. When solving problems involving composite figures, it is often necessary to calculate the area of each figure and then add these areas together.

For example, a figure is composed of a rectangle and two semi-circles. Determine the area of the figure.



Calculate the area of the rectangle.

$A = l \times w$

$A = (11)(5.5)$

$A = 60.5$ square inches

The two semi-circles together make one circle. Calculate the area of the circle.

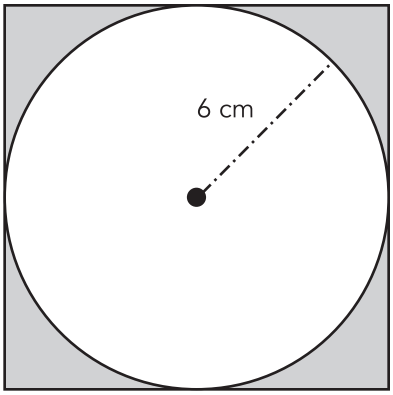
$A = \pi r^2$

$A = \pi(2.75)^2$

$A = 7.5625\pi \approx 23.75$ square inches

The area of the composite figure is approximately 60.5 square inches plus 23.75 square inches, or 84.55 square inches.

When determining the area of a shaded region of a figure, it is often necessary to calculate the area of a figure and subtract it from the area of a second figure.



For example, this figure shows a circle inscribed in a square. Determine the area of the shaded region.

When a circle is inscribed in a square, the diameter of the circle is equal to the side length of the square.

Calculate the area of the square.

$A = s^2$

$A = 12^2$

$A = 144$ square centimeters

Calculate the area of the circle.

$A = \pi r^2$

$A = \pi(6)^2$

$A = 36\pi \approx 113.04$ square inches

The area of the shaded region is approximately 144 square centimeters minus 113.04 square centimeters, or 30.96 square centimeters.